Cost analysis of warranty based on lemon law with multiple failures and total downtime

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Abstract

Analysis of warranty costs is a topic that is widely studied in various finished product industries. One type of warranty that applies is a product warranty under Lemon Law. This guarantee under Lemon Law applies a lot to automotive products such as cars. Lemon Law is a law that protects consumers from poor quality new goods that are not in accordance with related product standards but can reach consumers. With this Lemon Law, consumers can return products that are proven to have a lemon condition in the hands of producers to then get a refund (replacemen) or replacement with a new similar product (replacement). The product can be claimed in a lemon condition if: (1) the car has been returned to the dealer four times because it needs to be repaired on the same problem / faulty part, but the dealer is unable to fix it satisfactorily; or (2) the car has been out of service for more than 30 days due to one or more damage. Analysis of warranty costs in this study includes replacement cases with compliance with conditions (1) only, replacement cases with fulfillment of conditions (2) only, and replacement cases with fulfillment of conditions (1) and conditions (2). A simulation method is designed to illustrate variations in the condition of the damage to the car during the warranty period as well as variations in repair time when damage occurs based on a particular distribution. The algorithm of the simulation is then modified and developed to determine the expected cost that fulfill the conditions (1) and (2).

Keywords: Lemon law; simulation; replacement; downtime; cost expectations the warranty

INTRODUCTION

Lemon laws are laws aimed at protecting consumers against defective goods that do not conform to standards of quality and performance. The lemon laws provide the consumers the right to a new product or a full refund if the product is considered to be a lemon, and this in turn give more protection to the consumers. The state of Connecticut US became the first state to enact lemon laws for automobiles in 1982. In the next fiveyear time, 50 states and the District of Columbia had enacted the lemon laws protecting new-car buyers from defective automobiles. With these lemon laws, the consumers are allowed to return the defective car (which is lemon) to get refund or replacement with a new one, where before 1982 the consumers had bad experiences with frustration, delays, expense, and uncertainty to get the failed car fixed [1] Nowadays, the lemon laws have been adopted outside US – such as Canada, Europe, Australia, Singapore to name a few.

"An automobile is declared a lemon if either of two conditions are met: (i) the car has been returned to the dealer four times to have the same problem fixed, but the dealer was unable to repair the problem satisfactorily, or (ii) the car has been out of service more than 30 days due to one or more defects." Not all component failure results in a lemon. Only a critical component or system –e.g. gearbox, transmission, steering or braking systems, etc. covered under express warranty can cause a major defect or problem which influences car safety. Failure of the critical component or system is classified as a major defect as it is a safety-related problem. The lemon laws provide more protection to consumers to rectify repeat failures occurring during the warranty period. But the lemon laws require the car manufacturer to replace a defective car or refund its purchase price should the car is declared a lemon. As a results, the lemon laws result in additional cost to the manufacturer for rectifying a defective car, and this in turn affects the manufacturer's profits.

[2] studied automobile lemon laws to estimate the value of lemon protection to consumers. [3] examined tractor lemon laws and make comparison between automobile and tractor lemon laws. They used a principal agent to model the economic efficiency of lemon laws. In this paper, we study automobile lemon laws from the manufacture's view point and obtain the expected warranty servicing cost. From the manufacturer point of view, to obtain an accurate estimate of the servicing warranty cost for a car sold with warranty and protected by lemon laws, is an issue of great interest to manufacturers. As lemon laws give more burden (additional cost) to the manufacturer for servicing the lemon-law warranty.

The outline of the paper is as follows. In Section 2 we define lemon-law warranties and give the details of the model formulation. Two cases of a lemon-law warranty have been considered. Section 3 presents a numerical example for illustrating the estimate of the servicing warranty cost for the two lemon-law warranty cases. Finally, we conclude with a brief discussion of topics for future research in Section 4.

Model formulation

We consider that an item is sold with a one-dimensional warranty with warranty period W and the product is repairable. The product is protected by lemon laws which are enforceable *during the warranty period*. The product is declared a lemon if either of two conditions are met: (i) the car has been returned to the dealer k times to have the same problem fixed or (ii) the car has been out of service more than τ unit time (e.g., 30 days) due to one or more defects." We consider two cases – namely Cases 1 and 2.

Case 1: Look at the case where the lemon law only deals with number of failures and not the time out of action; and

Case 2: Take into account the number of failures as well as the downtime.

We assume that (i) the returned "lemon" is scrapped so there is no resale value to the manufacturer, and (ii) repair times are negligible.

Case 1: Lemon Law Warranty

In this case, the item is declared a 'lemon' if it fails k times during the warranty period. If the product is a "lemon", then the manufacturer has to refund the sale price to the customer or replace the failed item with a new item together with a new warranty policy at the time of the k^{th} failure.

As a results, for Case 1, we have two rectification actions - i.e. refund or replace the failed item. We first model Case 1 with refund and later on Case 1 with replacement.

Notations:

W	: Warranty period
C_p	: Item sale price
C_m	: Item manufacturing cost
C_r	: Average repair cost
C_c	: Collateral charges (incurred by the manufacturer if the item is declared a lemon under warranty)
$C(W;k)$	: The cost to the manufacturer to service the warranty
X_1	: The time to failure of the new item
$F(x)$ $f(x)$	: Distribution function and density function for X_1
$\lambda(x)$ $\Lambda(x)$	: Hazard function and cumulative hazard function associated with $F(x)$
X_j	: The operating time to the next item failure after $(j-1)$ repairs have been perfomed, $j \ge 1$
$S_n = \sum_{j=1}^{n} X_j$	: The time of the n^{th} failure, $n \ge 1$, has distribution function $F_n(x)$, density function $f_n(x)$ and survivor function $\bar{F}_n(x)$
$N(t)$	: The number of failures occuring in the interval $(0,t]$
$\Phi(z)$	: The standard normal distribution function

Case 1 (i): [Refund]

Customer starts to use the item at time $t = 0$, time instant of sale. If the *i*-th failure ($i<k$) under warranty is minimally repaired by the manufacturer at an average cost C_r. In this, the item is declared a lemon under warranty if the k^{th} failure occurs before W. If the item is a "lemon", then the manufacturer has to refund the total sale price to the customer should a critical component fail *k* times under warranty.

Let X_1 be the time to failure of the new item. X_1 has distribution function $F(x)$, density function $f(x)$, hazard function $\lambda(x)$ and cumulative hazard function $\Lambda(x)$.

Let X_j be the operating time to the next item failure after $(j-1)$ repairs have been performed $j \ge 1$. Define, $S_n = \sum_{j=1}^{n} X_j$, the time of the n^{th} failure, $n \ge 1$, has distribution function $F_n(x)$, density function $f_n(x)$ and survivor function $\bar{F}_n(x)$. If $N(t)$ is the number of failures that occur in the interval $(0,t]$ and each failure is fixed by a minimal repair, then $N(t)$ is a Nonhomogeneous Poisson process with intensity function $\lambda(t)$. The probability of *n* successive minimal repairs in $(0,t]$ is given by $\Pr\{N(t)=n\} = F_n(t) - F_{n+1}(t)$ where $P(S_n \le t) = F_n(t) = 1 - \sum_{i=0}^{n-1} e^{-\Lambda(t)} \Lambda(t)^i / i!$ [4]. A lemon is declared if the k^{th} failure occurs before W or if $S_k \le W$.

Warranty servicing cost:

Let $C_1(W;k)$ be the cost to the manufacturer to service the warranty for the case of refund. The warranty servicing cost $C_1(W;k)$ is $C_r N(W)$ if $S_k > W (\Leftrightarrow N(W) < k)$ and $(k-1)C_r + C_p + C_c$ if $S_k \le W$. Then, the expected warranty servicing cost is

$$\begin{aligned} E\left[C_1(W;k)\right] &= \sum_{n=0}^{k-1} nC_r P\{N(W)=n\} + [(k-1)C_r + C_p + C_c]P\{S_k \le W\} \\ &= C_r \sum_{n=1}^{k-1} n\left[F_n(W) - F_{n+1}(W)\right] + [(k-1)C_r + C_p + C_c]F_k(W) \qquad (1) \\ &= C_r \sum_{n=1}^{k-1} F_n(W) + (C_p + C_c)F_k(W) \end{aligned}$$

The second moment of the warranty servicing cost is

$$\begin{aligned} E\left[C_1^2(W;k)\right] &= \sum_{n=1}^{k-1} (nC_r)^2 \left[F_n(W) - F_{n+1}(W)\right] + [(k-1)C_r + C_p + C_c]^2 F_k(W) \\ &= C_r^2 \sum_{n=1}^{k-1} (2n-1) F_n(W) + (C_p + C_c)\left[C_p + C_c + 2(k-1)C_r\right]F_k(W) \end{aligned} \qquad (2)$$

The variance of the warranty servicing cost is

$$Var\left[C_1(W;k)\right] = E\left[C_1(W;k)^2\right] - \left(E\left[C_1(W;k)\right]\right)^2 \qquad 3)$$

Let C_L be the warranty servicing cost limit. If $C_1(W;k)$ is normally distributed, then the probability that $C_1(W;k)$ will exceed some defined limit, C_L is given by

$$\Pr\{C_1(W;k) > C_L\} = 1 - \Phi\left(C_L - E\left[C_1(W;k)\right] \Big/ \sqrt{Var\left[C_1(W;k)\right]}\right), \qquad 4)$$

where $\Phi(z)$ is the standard normal distribution function.

Case 1 (ii): [Replacement with a new warranty]

Here, if the item is declared a lemon under warranty (or $S_k \le W$), then the failed item is replaced with a new item with a new warranty, and hence have a renewing warranty. Let $C_2(W;k)$ be the cost to the manufacturer to service the warranty for the case of replacement. We obtain $E[C_2(W;k)]$ by conditioning on S_k the time of the k^{th} failure.

$$E\left[C_2(W;k) | S_k = s\right] = \begin{cases} E\left[N(W) | N(W) < k\right]C_r & if\ s > W \\ (k-1)C_r + C_m + C_c + E\left[C_2(W;k)\right] & if\ s \le W \end{cases} \qquad 5)$$

Removing the conditioning gives

$$\begin{aligned} E\left[C_2(W;k)\right] &= \sum_{n=0}^{k-1} nC_r P\{N(W)=n\} \\ &+ \left[(k-1)C_r + C_m + C_c + E\left[C_2(W;k)\right]\right]F_k(W) \qquad (6) \\ &= C_r \sum_{n=1}^{k-1} F_n(W) + (C_m + C_c)F_k(W) + E\left[C_2(W;k)\right]F_k(W), \end{aligned}$$

Then

$$E\left[C_2(W;k)\right] = C_r \sum_{n=1}^{k-1} F_n(W) + (C_m + C_c)F_k(W) \Big/ \bar{F}_k(W) \qquad 7)$$

Note: $E[C_2(W;1)] = (C_m + C_c)F(W) / \bar{F}(W)$ is the expected servicing cost to the manufacturer of a renewing warranty where the item is replaced by a new one if it fails before W. Using a similar conditioning argument,

$$E\left[C_2^2(W;k)\middle|S_k=s\right]=\begin{cases}E\left[(N(W)|N(W)<k)^2\right]C_r^2 & if\ s>W\\ E\left[((k-1)C_r+C_m+C_c+C_2(W;k))^2\right] & if\ s\le W\end{cases} \tag{8}$$

Removing the conditioning gives

$$E\left[C_2^2(W;k)\right]=$$

$$=\left.\begin{Bmatrix}C_r^2\bar{F}_k(W)\sum_{n=1}^{k-1}\begin{bmatrix}(2n-1)F_n(W)\\ +2C_r\begin{bmatrix}(k-1)C_r\\ +C_m+C_c\end{bmatrix}F_k(W)\sum_{n=1}^{k-1}F_n(W)\end{bmatrix}\\ +(C_m+C_c)\left[C_m+C_c+2(k-1)C_r\right]F_k(W)\\ +(C_m+C_c)^2F_k(W)^2\end{Bmatrix}\middle/\bar{F}_k(W)^2\right. \tag{9}$$

Variance of $C_2(W;k)$ is given by

$$Var\left[C_2(W;k)\right]=E\left[C_2(W;k)^2\right]-\left(E\left[C_2(W;k)\right]\right)^2$$

10)

The probability that $C_2(W;k)$ will exceed some defined limit, C_L is given by (4) with $E[C_2(W;k)]$ and $Var[C_2(W;k)]$ given in (7) and (10).

Case 2: Lemon Law Warranty

An item is declared a 'lemon' if it fails k times during the warranty period or if the total time taken to repair the item (total downtime) under warranty exceeds τ. If the lemon law is invoked by one of these two events, the manufacturer refunds the sale price to the customer [Case2-(i)] or replaces the failed item with a new item together with a new warranty policy [Case2-(ii)]. Let $Y_1,Y_2,Y_3,\ldots$ be the successive item repair times. $Y_1,Y_2,Y_3,\ldots$ are *iid* random variables with distribution function $G(y)$. Define, $R_n=\sum_{j=1}^{n}Y_j$ is the sum of the first n repair time $n\ge 1$. The distribution function $G_n(y)=\Pr\{R_n\le y\}$ is the n-fold convolution of G with itself $[G_0(y)\equiv 1]$; $g_n(y)=dG_n(y)/dy$ is the corresponding density function. Let S_k be the time when the k^{th} failure occurs and the total downtime up to this point does not exceed τ [S_k occurs at a time instance of failure]. If L_τ is the time when the total downtime first exceeds τ and less than k failures have occurred up to this point [L_τ occurs during a repair period], then $L=\min(S_k,L_\tau)$.

As a result, the item is declared a lemon under warranty if and only if $L=\min(S_k,L_\tau)$, $L\le W$. Now we obtain the probability that an item is declared a lemon, Pr{item declared a lemon under warranty} $L\le W=\Pr\{L\le W\}=1-\Pr\{L_k>W\}\Pr\{L_\tau>W\}$. We need to find the distribution functions $\Pr\{L_\tau\le z\}$ and $\Pr\{L_k\le z\}$.

Case 2 (i): [Refund]

An item is a "lemon", if an item fail k times under warranty (or $L=S_k\le W$) or the total repair time of the item (total downtime) under warranty exceeds τ. (or $L=L_\tau\le W$), whichever comes first. Here, when the item is declared a lemon then the manufacturer has to refund the total sale price to the customer.

Case 2 (ii): [Replacement with a new warranty]

Here, if the item is declared a lemon under warranty, then the failed item is replaced with a new item with a new warranty, and hence have a renewing warranty.

(Note: the mathematical formulations for these cases are more involved and hence leave as future works)

Numerical example

We first consider that the product failure given by Weibull distribution with $F(t)=1-\exp(-t/\alpha)^{\beta}$ where α and β are the scale and shape parameters, respectively, and then a mixture Weibull distribution. We use the following parameter values: C_p = 100, C_c = 0.05C_p, C_m = 0.7C_p, C_r = 0.05C_p, C_L = 0.3 C_p , W=1 and k = 4 with $\beta = 2$.

Table 1. Results for refund and replacement cases

α	$E[T]$	Case 1 (i) Refund Case		Case 1(ii) Replacement Case	
		$\frac{E[C(W,k)]}{C_p}$	$P\{C(W,k)>C_L\}$	$\frac{E[C(W,k)]}{C_p}$	$P\{C(W,k)>C_L\}$
0.99	0.87736	0.07104	0.143307	0.07148	0.10839
1.01	0.89509	0.06662	0.131022	0.06696	0.09860
1.20	1.06347	0.04028	0.062993	0.04031	0.05089
1.30	1.15209	0.03276	0.048590	0.03277	0.04177
1.40	1,24072	0.02738	0.040530	0.02739	0.03661

Table 1 shows $E[C(W,k)]/C_p$ and $P\{C(W,k)>C_L\}$ for α =0.99 to 1.40 (Note that greater α means higher the reliability of the product). For refund case, the expected warranty servicing cost decreases from 7 to 2.7% of the purchase price when α increases from 0.99 to 1.40. This is as expected since higher reliability of the product will lower the expected warranty servicing cost. When C_L = 0.3C_p , the risk of the warranty servicing cost exceeds C_L is 10.8% and it decreases with the increase of α. For replacement case, the effect of reliability to the expected warranty servicing cost and the risk of the burden exceeds C_L is similar. There is no much different between the expected warranty servicing cost of the refund and that of replacement for each value of α . The pattern holds for the risk of the burden exceeds C_L.

Table 2. Results for refund and replacement cases with $P\{C(W,k)>C_L\}=0.05$

α	$E[T]$	Case 1 (i) Refund Case		Case 1(ii) Replacement Case	
		$E[C(W,k)]/C_p$	C_L/C_p	$E[C(W,k)]/C_p$	C_L/C_p
0.99	0.87736	0.071036	0.4255758	0.071476	0.3767627
1.01	0.89509	0.066619	0.4099569	0.066958	0.3651404
1.20	1.06347	0.040277	0.3203483	0.040309	0.3021915
1.30	1.15209	0.032762	0.2986045	0.032773	0.2875691
1.40	1.24072	0.027384	0.2852304	0.027388	0.2784766

Table 2 shows results for C_L/C_p when $P\{C(W,k)>C_L\}$ is fixed (i.e. 0.05). If the risk is kept constant then C_L/C_p for refund is slightly higher than that of replacement. We now consider a situation where the population of the products comprising of conforming items and non-conforming items and hence the failure distribution for the item is given by the a mixture Weibull distribution with $F(t;\alpha)=pF_c(t;\alpha_c)+(1-p)F_{nc}(t;\alpha_{nc}), \alpha_c>\alpha_{nc}$ where $F_c(t;\alpha_c)$ and $F_{nc}(t;\alpha_{nc})$ are the failure distributions for the conforming and non-conforming items with scale parameters α_c , α_{nc} , respectively.

Table 3. Results for a mixture Weibull distribution with p = 0.9, β = 2, $\alpha_{nc}=0.65$

α_c	Case 1 (i) Refund Case		Case 1(ii) Replacement Case	
	$E[C(W,k)]/C_p$	$P\{C(W,k)>C_L\}$	$E[C(W,k)]/C_p$	$P\{C(W,k)>C_L\}$
0.9	0.114627	0.252635	0.117169	0.214032

α_c	Case 1 (i) Refund Case		Case 1(ii) Replacement Case	
	$E[C(W,k)]/C_p$	$P\{C(W,k)>C_L\}$	$E[C(W,k)]/C_p$	$P\{C(W,k)>C_L\}$
1.0	0.084993	0.181203	0.085873	0.141194
1.1	0.066639	0.131079	0.066979	0.098649
1,2	0.054621	0.098054	0.054766	0.074236
1.3	0.046343	0.076867	0.046410	0.059875

Table 3 shows results for the case where reliability of the conforming item improves and this results in the decreasing in the expected warranty servicing cost for both cases. If p decreases meaning that there is some improvement in a production process (from 20% to 1% of on conforming items), then the expected warranty servicing cost decreases – from 6.3% to 3.39% for the refund case and from 6.33% to 3.4% for the replacement case (there are no much different) (see Table 4).

Table 4. Results for a mixture Weibull distribution with α_0 = 1.3, α_1 = 0.65 and $p = 0{,}8,\ldots,0.99$

p	Case 1 (i) Refund Case		Case 1(ii) Replacement Case	
	$E[C(W,k)]/C_p$	$P\{C(W,k)>C_L\}$	$E[C(W,k)]/C_p$	$P\{C(W,k)>C_L\}$
0.80	0.063061	0.121113	0.063331	0.091007
0.90	0.046343	0.076867	0.046410	0.059875
0.95	0.039202	0.060727	0.039230	0.049456
0.97	0.036547	0.055414	0.036567	0.046087
0.99	0.033999	0.050719	0.034012	0.043118

CONCLUSIONS

This paper deals with estimating warranty servicing cost for a product sold with warranty where lemon laws are enforceable during the warranty period.

We studied two cases – refund and replacement cases where an item is declared a lemon only if it fails k times during the warranty period. In general, an item turns out to be a lemon either (i) it fails k times during the warranty period or (ii) the total time taken to repair the item (total downtime) under warranty exceeds τ, whichever comes first. This has been indicated earlier as one further research topic. Other future research topics are as follows. One can study the case where the customer can either choose (i) refund or (ii) replacement, and the customer's choice of (i) or (ii) occurs randomly. This scenario is similar with that of the combination of FRW and PRW for 2-d warranties in [5]and [6]. Also, this lemon law warranty studied can be extended to two dimensional warranties and these topics are under investigation.

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